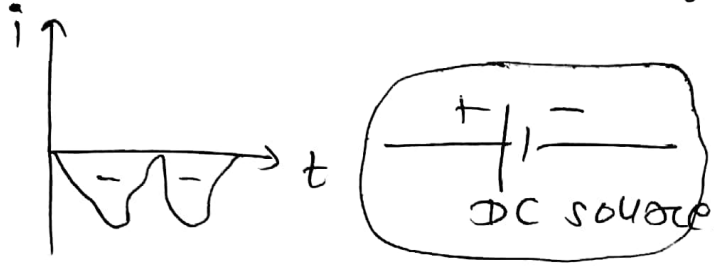
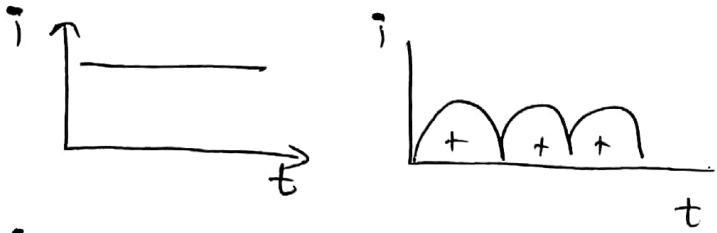


Alternating current (AC) :-

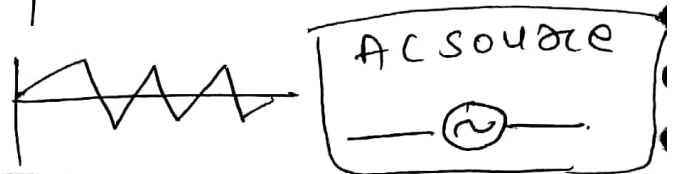
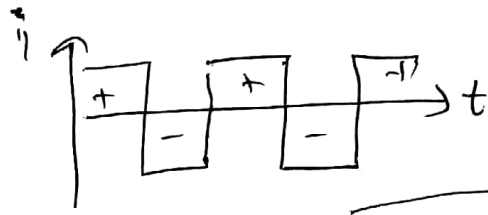
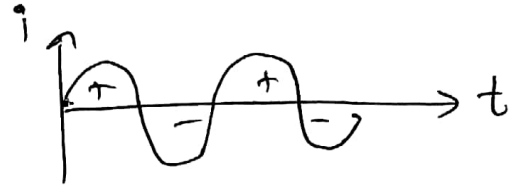
* Direct current :-

- flows in one direction.
- flows only in one direction



* Alternating current :-

- This will in both direction.
- (i.e. changes its direction)



* If any quantity depends on time or it's function of time \Rightarrow average value is given by :-

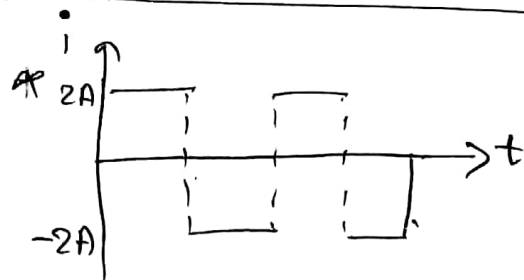
i.e. $y = f(t)$
 $\therefore y_{avg} = \frac{\int y dt}{\int dt}$

→ $i_{avg} = \frac{\int i dt}{\int dt}$

$$= 3 \left(\frac{t^3}{3} \right)_0^2 / (t)_0^2$$

$$= (8 - 0) / (2 - 0)$$

$$= \underline{4A}$$



Q.1) If $i = 3t^2$, find avg i in 2sec.

~~$i = 3(2)^2$
 $= 3(4)$
 $= 12$~~

$$i_{avg} = \frac{\int_0^2 i dt}{\int_0^2 dt}$$

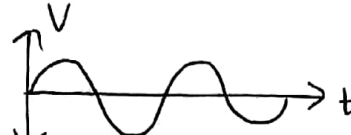
$$= \frac{\int_0^2 3t^2 dt}{\int_0^2 dt}$$

⊙ $i_{avg} = \langle i \rangle = \bar{i} = 0$
 (full cycle)

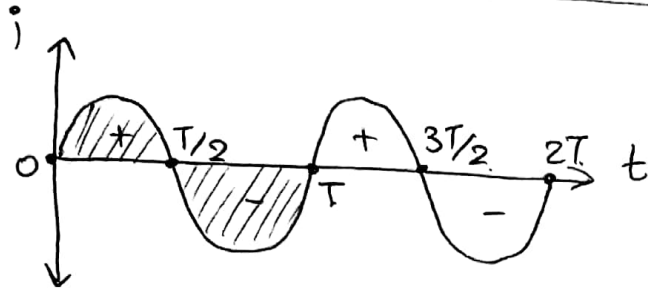
⊙ $i_{avg} = |\langle i \rangle| = 2A$
 (Half cycle)

⊙ $\langle i^2 \rangle = \bar{i^2} = 4A$

AC generator :-

$$V = V_0 \sin \omega t$$


$$i = i_0 \sin \omega t$$

(Here: $i_{avg} = 0$
(full cycle))

proof:-

$$i_{avg} = \frac{\int_0^T i dt}{\int_0^T dt}$$

$$= \frac{\int_0^T i_0 \sin \omega t dt}{\int_0^T dt}$$

$$= \frac{i_0 \left(-\frac{\cos \omega t}{\omega} \right)_0^T}{T}$$

$$= \frac{-i_0 (\cos \omega T - \cos 0)}{\omega T}$$

$$= \frac{-i_0 (\cos \omega T - 1)}{\omega T}$$

$$= \frac{-i_0 \left(\cos \omega \left(\frac{2\pi}{\omega} \right) - 1 \right)}{\omega T}$$

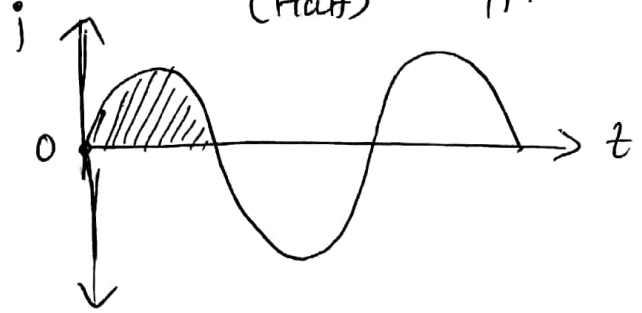
$$= \frac{i_0}{\omega T} (+1 - 0)$$

$$i_{avg} = \underline{\underline{0}}$$


* Find average value of (2) AC current in half cycle.

$$i_{avg} = \frac{2i_0}{\pi}$$

(Half)



Ans:-

$$i_{avg} = \frac{\int_0^{\pi/2} i dt}{\int_0^{\pi/2} dt}$$

$$= \frac{\int_0^{\pi/2} i_0 \sin \omega t dt}{\left(t \right)_0^{\pi/2}}$$

$$= \frac{i_0 \left(-\frac{\cos \omega t}{\omega} \right)_0^{\pi/2}}{\left(\frac{\pi}{2} \right)}$$

$$= \frac{-i_0 \left[\cos \omega \frac{\pi}{2} - \cos 0 \right]}{\frac{\pi}{2}}$$

$$= \frac{-2i_0 \left[\cos \omega \frac{\pi}{2} - 1 \right]}{\pi \omega}$$

$$= \frac{-2i_0 [-1 - 1]}{\pi \omega}$$

$$= \frac{-2i_0 (-2)}{\frac{\pi}{\omega}}$$

$$i_{avg} = \frac{+2i_0}{\pi} \quad \frac{1}{2} \text{ cycle.}$$

Similarly:

$$V_{avg} = \frac{+2V_0}{\pi} \quad \text{for } \frac{1}{2} \text{ cycle.}$$

RMS value: (Root mean square)

- 1) do square
- 2) do average (mean)
- 3) do root.

$$i_{rms} = \sqrt{\langle i^2 \rangle}$$

$$V_{rms} = \sqrt{\langle v^2 \rangle}$$

Main Formula.

Sinusoidal

$$V = V_0 \sin \omega t \Rightarrow V_{rms} = \frac{V_0}{\sqrt{2}}$$

$$i = i_0 \sin \omega t \Rightarrow i_{rms} = \frac{i_0}{\sqrt{2}}$$

* Prove: For sinusoidal current $i = i_0 \sin \omega t$; ~~average~~
rms value of curr. is
 $i_{rms} = \frac{i_0}{\sqrt{2}}$

Ans:- $i_{rms} = \sqrt{\langle i^2 \rangle}$

$$= \sqrt{\langle i_0^2 \sin^2 \omega t \rangle}$$

$$= \left(\frac{\int_0^T i_0^2 \sin^2 \omega t \, dt}{\int_0^T dt} \right)^{1/2}$$

$$= \left(i_0^2 \int_0^T \frac{\sin^2 \omega t \, dt}{T} \right)^{1/2}$$

$$= \left(\frac{i_0^2}{T} \int_0^T \left(\frac{1 - \cos 2\omega t}{2} \right) dt \right)^{1/2}$$

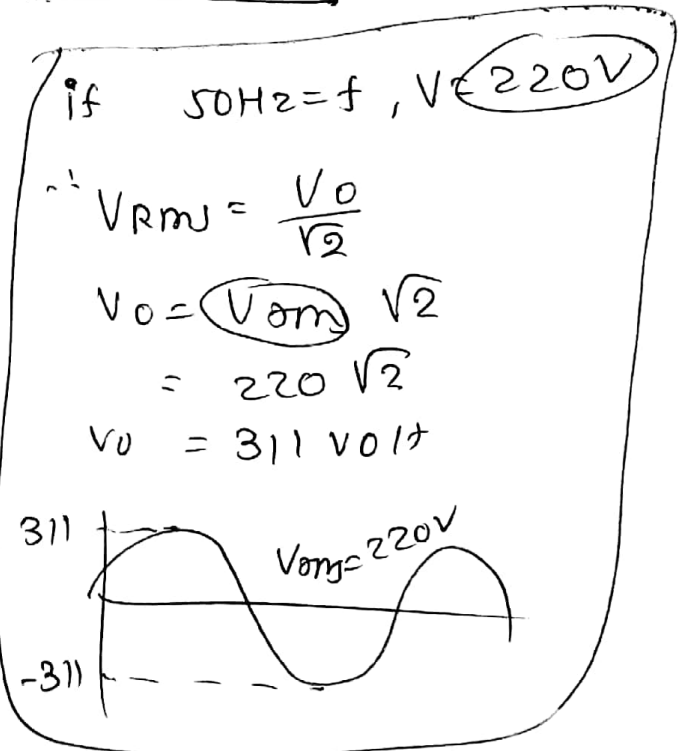
$$= \left(\frac{i_0^2}{2T} \left(t - \frac{\sin 2\omega t}{2\omega} \right) \Big|_0^T \right)^{1/2} \Rightarrow V_{Rms}$$

$$i_{rms} = \left[\frac{i_0^2}{2T} \left(T - \frac{\sin 4\omega T}{2\omega} - 0 + 0 \right) \right]^{1/2}$$

$$= \left(\frac{i_0^2}{2T} \left[T - \frac{\sin 4\pi}{2\omega} \right] \right)^{1/2}$$

$$= \left(\frac{i_0^2}{2T} [T - 0] \right)^{1/2}$$

$$i_{rms} = \frac{i_0}{\sqrt{2}}$$

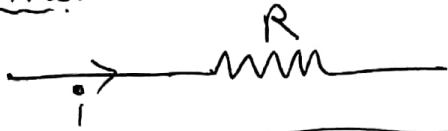


V_0 = peak voltage.

• If in exam not given what kind of V is?
 V_{avg} , V_0 , V_{Rms} ?

$$\Rightarrow V_{Rms}$$

* Energy dissipated in a resistor by current $i = i_0 \sin \omega t$ in time t is same as Energy diss. by a constant current i_{rms} .



Dissipated Energy

$$E = i_{rms}^2 R t$$

Proof:- Energy dissipated in small time dt :

$$\int dE = \int i^2 R dt$$

$$\therefore E = R \int i^2 dt$$

$$\left(\begin{aligned} i_{rms} &= \sqrt{\langle i^2 \rangle} \\ &= \sqrt{\frac{\int i^2 dt}{\int dt}} \end{aligned} \right)$$

★ Suppose, Here i_{rms} (const. curr.) flows.

* To measure AC current, AC voltage,

→ Hot wire Voltmeter & current meter are used

They measure RMS value.

⇒ we are not able to measure AC with DC devices.

(4)

Q/A:- Find RMS for:
 $i = i_0 + i_0 \sin \omega t$
 $i_{rms} = \sqrt{\langle i^2 \rangle} = \left(\frac{\int i^2 dt}{\int dt} \right)^{1/2}$

Shortcut:- (for full cycle)

$$\langle \sin \omega t \rangle = \overline{\sin \omega t} = 0$$

$$3) \langle \cos \omega t \rangle = \overline{\cos \omega t} = 0$$

$$3) \langle \sin^2 \omega t \rangle = 1/2$$

$$4) \langle \cos^2 \omega t \rangle = 1/2$$

Q/B Find RMS i for: $i = i_0 \sin \omega t$.

$$\begin{aligned} i_{rms} &= \sqrt{\langle i^2 \rangle} = \sqrt{\langle i_0^2 \sin^2 \omega t \rangle} \\ &= \sqrt{\langle i_0^2 \rangle \langle \sin^2 \omega t \rangle} \\ &= \sqrt{\langle i_0^2 \rangle} \frac{1}{\sqrt{2}} \\ &= \frac{i_0}{\sqrt{2}} \end{aligned}$$

Ans. A: $i_{rms} = \sqrt{\langle i^2 \rangle}$

$$\begin{aligned} &= \sqrt{i_0^2 + i_0^2 \sin^2 \omega t + 2i_0^2 \sin \omega t} \\ &= \sqrt{i_0^2 + i_0^2 \sin^2 + 2i_0^2 \sin} \\ &= \sqrt{i_0^2 + \frac{i_0^2}{2} + 2i_0^2 \sin} \\ &= i_0 \sqrt{\frac{3}{2}} \end{aligned}$$

Q/C: $i = i_1 \sin \omega t + i_2 \cos \omega t$
(Home work)

Ans: $\sqrt{\frac{i_1^2 + i_2^2}{2}}$

Hint: $2 \sin \theta \cos \theta = \sin 2\theta$

$2 \sin \omega t \cos \omega t = \sin 2\omega t$

* What is Phase?

$$V = V_0 \sin(\omega t) ; i = i_0 \sin(\omega t)$$

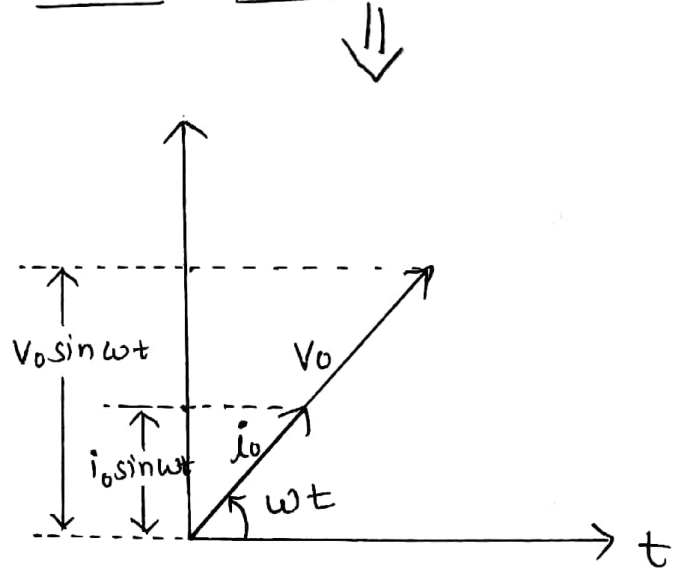
↙ Phase ↘

* Phase difference = $\left| \begin{array}{l} \text{Phase of } V \\ \text{Phase of } i \end{array} \right|$

= $|\omega t - \omega t| = 0$

Here; current i in phase with Voltage. (S)

PHASOR DIAGRAM:-



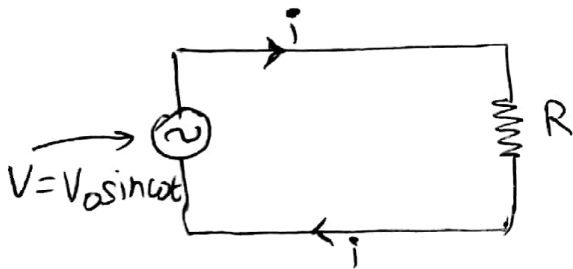
→ Here; i_0 is small because ($i_0 = V_0/R$).

→ Length of Arrow → Peak Value

→ Projection on Y-axis → instantaneous
 $V = V_0 \sin \omega t$
 $i = i_0 \sin \omega t$

* CIRCUIT THEORY:-

① Pure Resistive circuit



• Applying K-VL: $\oint E \cdot dl = 0$

$$\therefore V - iR = 0$$

$$\therefore V = iR$$

$$\therefore i = \frac{V}{R} = \frac{V_0 \sin \omega t}{R}$$

$$\therefore i = \left(\frac{V_0}{R}\right) \sin \omega t$$

$$\therefore \boxed{i = (i_0) \sin \omega t}$$

where; ($i_0 = \frac{V_0}{R}$); $i_0 = \text{Peak } I$
 $V_0 = \text{Peak } V$.

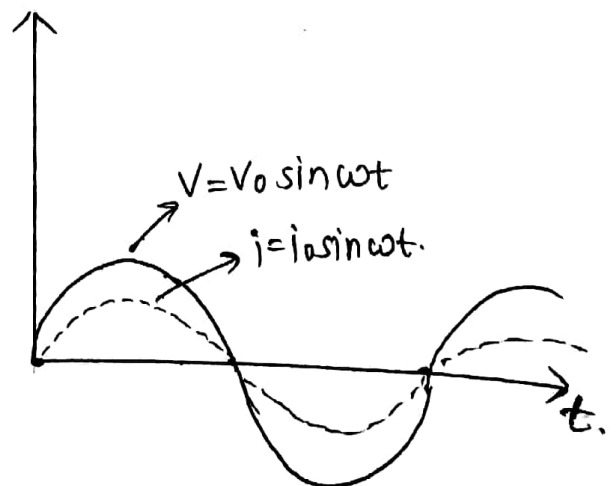
• Phase difference = $\left| \begin{array}{l} \text{Phase of } V \\ \text{Phase of } i \end{array} \right|$

$$= |\omega t - \omega t|$$

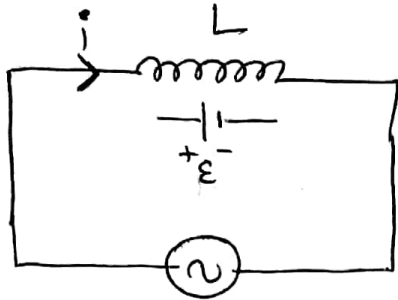
$$= 0.$$



WAVE DIAGRAM:-



②. Pure Inductive CKT:-



$$V = V_0 \sin \omega t$$

Here, inductor opposes the change so it becomes battery (produce in opposite direction) of emf $\epsilon = L \frac{di}{dt}$

$$\therefore V - \epsilon = 0$$

$$\therefore V - L \frac{di}{dt} = 0$$

$$\therefore di = \frac{V}{L} dt = \frac{V_0 \sin \omega t}{L} dt$$

$$\therefore \int di = \frac{V_0}{L} \int \sin \omega t dt$$

$$\therefore i = \frac{V_0}{L} \left(-\frac{\cos \omega t}{\omega} \right)$$

$$\therefore i = -\frac{V_0}{\omega L} \cos \omega t$$

$$\boxed{i = -i_0 \cos \omega t}$$

where: $i_0 = \frac{V_0}{\omega L} = \frac{V_0}{X_L}$

$X_L = \omega L$ = Inductive Reactance
(its like resistance of Inductor.)

unit $\rightarrow \Omega$.

$$\therefore \sin(90^\circ - 0) = 1 \text{ or } 0$$

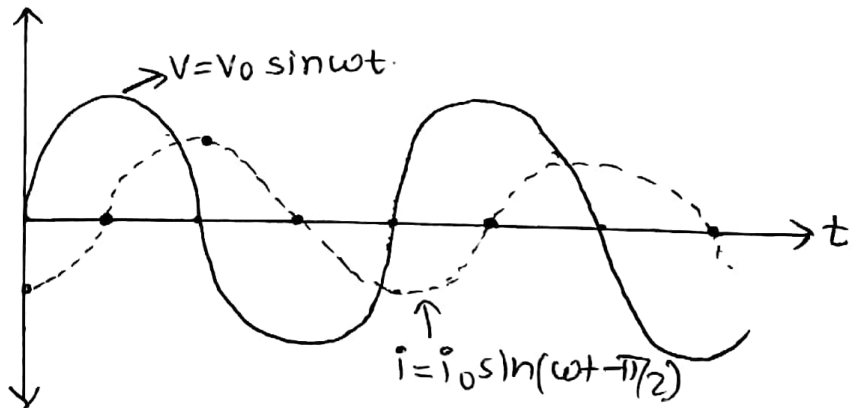
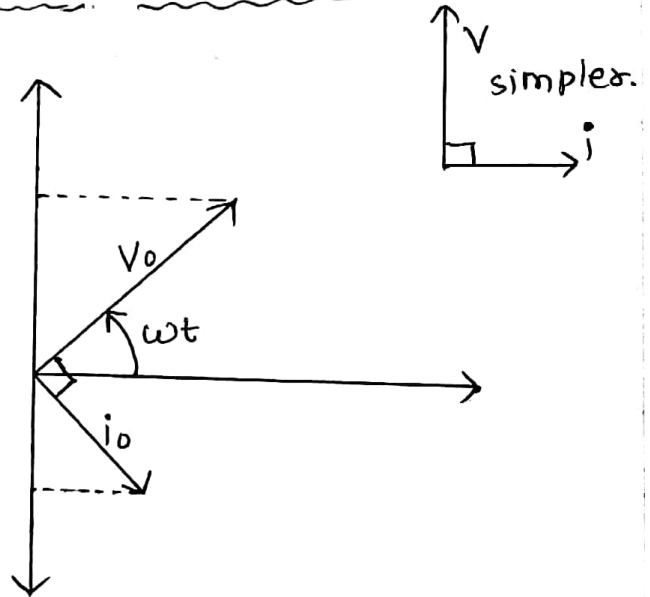
$$\therefore i = -i_0 \sin(90^\circ - \omega t)$$

$$\therefore i = -i_0 \left(\sin \left(\frac{\pi}{2} - \omega t \right) \right)$$

$$\therefore \boxed{i = i_0 \sin(\omega t - \pi/2)}$$

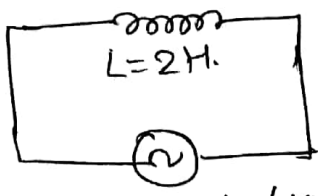
"Current lags behind Potential by 90° "

PHASOR DIAGRAM :-



max \rightarrow min \rightarrow Pot \rightarrow min.

Q:-
A



$V = 10 \sin(10t + 30)$

Find Eqⁿ of current v/s time
Also $i_{rms} = ?$

$V = V_0 \sin(\omega t + \phi)$

\therefore compare with
 $V = 10 \sin(10t + 30)$

$\therefore V_0 = 10, \omega = 10, \phi = 30$

\rightarrow Now $i_0 = \frac{V_0}{X_L} = \frac{10}{(\omega L)(2)} = \frac{0.5 A}{\omega L}$

\rightarrow $i_{rms} = \frac{i_0}{\sqrt{2}} = \frac{0.5}{\sqrt{2}}$

\rightarrow current lags behind voltage by 90°

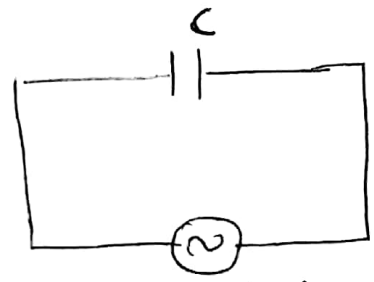
$\therefore i = 0.5 \sin(10t + 30 - 90)$
 $i = 0.5 \sin(10t - 60)$

Error:- $i = \frac{V}{X_L}$ (X) \rightarrow $i \neq V$ use $v = i \omega L$
we can't use this Eqⁿ
we can use $i_0 = \frac{V_0}{X_L}$

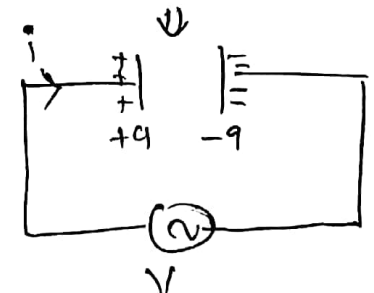
For const. current / DC current
 $f = 0$
 $\omega = 0$
 $X_L = 0$
i.e in DC Batteries NO Inductive Reactance produced.

Q B :- Find Inductive Reactance of an inductor $L = 2H$ connected to an AC freq. of 50Hz.
 $X_L = \omega L = 2\pi f L = 2\pi \times 50 \times 2$
 $X_L = 200\pi \Omega$

③ PURE CAPACITIVE CIRCUIT :-



$V = V_0 \sin \omega t$



$\therefore V - \frac{q}{C} = 0$ (K-V-L)

$\therefore q = CV$

$\therefore q = C V_0 \sin \omega t$

$\therefore \frac{dq}{dt} = C V_0 \frac{d}{dt} (\sin \omega t)$

$\therefore i = C V_0 \cos \omega t \times \omega$

$i = (\omega C V_0) \cos \omega t$

$i = (i_0) \cos \omega t$

$X_L = \omega L$
 $X_L = (2\pi f) L$ ($\because \omega = 2\pi f$)



$f \uparrow \Rightarrow X_L \uparrow \Rightarrow i_0 \downarrow$

let: $i_0 = \omega C V_0 = \frac{V_0}{(1/\omega C)}$

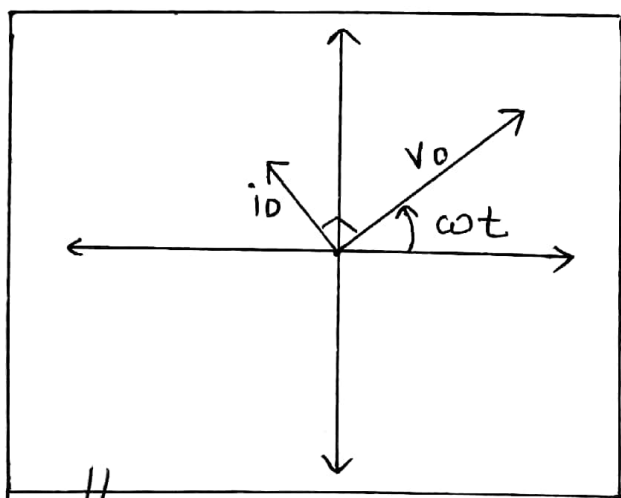
where: $\left(\frac{1}{\omega C} = X_C = \text{capacitive Reactance}\right)$

$i = i_0 \cos \omega t$

$i = i_0 \sin(\omega t + \pi/2)$

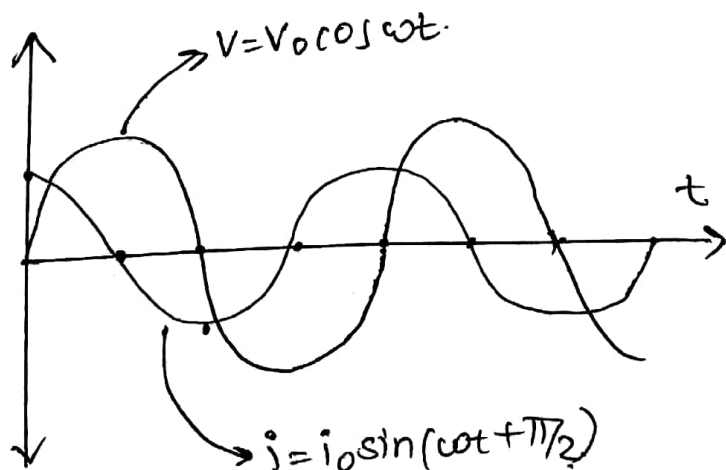
"current leads Potential by 90° ."

PHASOR :-



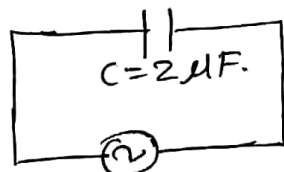
Simply \Rightarrow $i \rightarrow$ v

wave diagram :-



$I \rightarrow \max \Rightarrow V \rightarrow \min$ vice versa

Q A :- Find Equatⁿ of I v/s t .
calculate i_{avg} for
i) Full cycle ii) Half cycle.



$V = 10 \sin 10t$

Here: $V_0 = 10, \omega = 10$

$\therefore i_0 = \frac{V_0}{X_C} = \frac{V_0}{1/\omega C} = \frac{V_0}{1/10 \times 2 \times 10^{-6}}$

$\therefore i_0 = 10(10 \times 2 \times 10^{-6}) = 200 \times 10^{-6}$

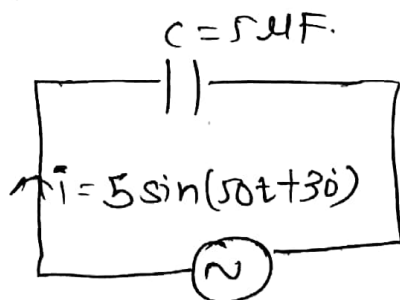
$i_0 = 2 \times 10^{-4} A$

i) Full cycle $i = 0$

ii) Half $i = \frac{2i_0}{\pi} = \frac{2 \times 2 \times 10^{-4}}{\pi}$

(Here; also $i(t) = \frac{V(t)}{X_C}$ not possible bco its not in phase ($V \neq i$))

Q B :-



$V = ?$

Find Equⁿ of V v/s time

i is 90° ahead to V

i.e V lags

$\therefore V = V_0 \sin(50t + 30^\circ - 90^\circ) = V_0 \sin(50t - 60^\circ)$

$\therefore V_0 = i_0(X_C) = 5 \left(\frac{1}{\omega C}\right) = 5 \left(\frac{1}{50 \times 5 \times 10^{-6}}\right)$

Pure Resistive

$$V = V_0 \sin \omega t$$

$$i = i_0 \sin \omega t$$

$$i_0 = \frac{V_0}{R}$$



~~R is independent~~
R is independent of frequency.

Pure Inductive

$$V = V_0 \sin \omega t$$

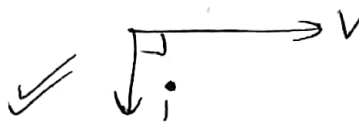
$$i = i_0 \sin(\omega t - \pi/2)$$

$$i_0 = \frac{V_0}{X_L}$$

$$X_L = 2\pi fL = \omega L$$



or



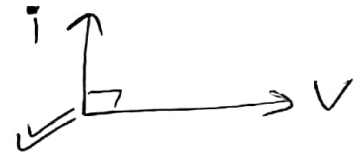
Pure Capacitive ⁽⁹⁾

$$V = V_0 \sin \omega t$$

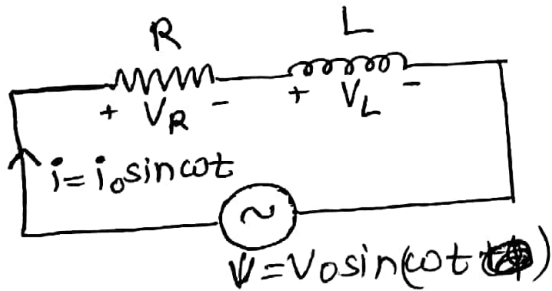
$$i = i_0 \sin(\omega t + \pi/2)$$

$$i_0 = \frac{V_0}{X_C}$$

$$X_C = \frac{1}{\omega C}$$

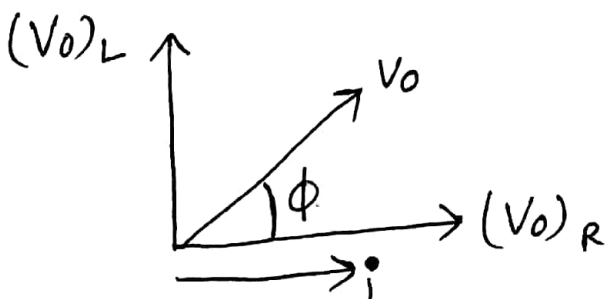


① Series L-R circuit :-



$$V_R(t) = (V_0)_R \sin \omega t$$

$$V_L(t) = (V_0)_L \sin(\omega t + \pi/2)$$



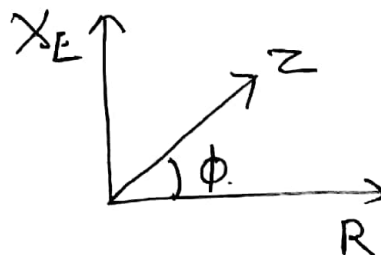
$$\therefore V_0^2 = (V_0)_R^2 + (V_0)_L^2$$

$$\therefore V_0 = \sqrt{V_{0R}^2 + V_{0L}^2}$$

$$= \sqrt{i_0^2 R^2 + i_0^2 X_L^2}$$

$$V_0 = i_0 \sqrt{X_L^2 + R^2}$$

$$V_0 = i_0 \times Z$$



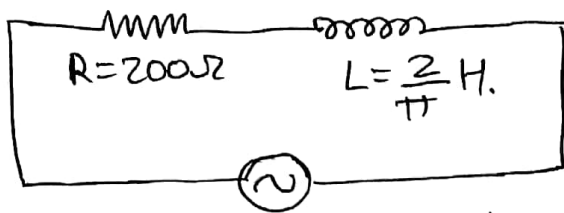
$$i.e. \quad Z = \sqrt{R^2 + X_L^2}$$

$$\tan \phi = \frac{X_L}{R}$$

i.e. $|Z| = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \omega^2 L^2}$

$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{\omega L}{R}\right)$

Q A



$V = 200 \sin 100\pi t$

Find i) inductive Reactance X_L

ii) impedance.

iii) Peak current i_0 .

iv) $i(t) = ?$

Ans i) $X_L = \omega L = (100\pi) \frac{2}{\pi} = 200 \Omega$

ii) $Z = \sqrt{R^2 + X_L^2} = \sqrt{200^2 + 200^2}$
 $= \sqrt{200(400)}$
 $= 20\sqrt{100 \times 2}$
 $= 20 \times 10 \times \sqrt{2}$
 $= 200\sqrt{2}$

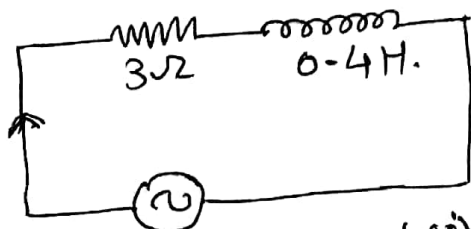
iii) $i_0 = \frac{V_0}{Z} = \frac{200}{282} = 0.707$

iv) $i(t) = i_0 \sin(\omega t - \phi)$
 $i(t) = (0.707) \sin(100\pi t - \phi)$

where; $\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{200}{200}\right)$
 $= 45^\circ$

$\therefore i(t) = 0.707 \sin(100\pi t - 45^\circ)$

Q B



$E = 20 \sin(10t + 60^\circ)$

Find. i) X_L ii) Z iii) i_0

iv) $i(t)$ v) $V_R(t)$ & $V_L(t)$

$V_0 = 20V, \omega = 10, \phi = 60^\circ$ (10)

i) $X_L = \omega L = 10(0.4) = 4 \Omega$

ii) $Z = \sqrt{R^2 + X_L^2} = \sqrt{9 + 16} = 5$

iii) $i_0 = \frac{V_0}{Z} = \frac{20}{5} = 4A$

iv) $i(t) = i_0 \sin(\omega t + \phi - \phi)$

$\rightarrow \phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{4}{3}\right)$

$(\phi = 53^\circ)$

$\therefore i(t) = 4 \sin(10t + 60^\circ - 53^\circ)$

$[i(t) = 4 \sin(10t + 7^\circ)]$

v) $V_R(t) = V_{0R} \sin(10t + 7^\circ)$
 $= (i_0 \times R) \sin(10t + 7^\circ)$
 $= (4 \times 3) \sin(10t + 7^\circ)$

$V_R(t) = 12 \sin(10t + 7^\circ)$

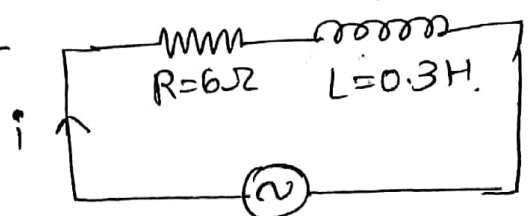
vi) $V_L(t) = V_{0L} \sin(10t + 97^\circ)$

$= (i_0 \times X_L) \sin(10t + 97^\circ)$

$= (4 \times 4) \sin(10t + 97^\circ)$

$V_L(t) = 16 \sin(10t + 97^\circ)$

Q C



$i = 10 \sin(20t + 30^\circ)$

Find i) X_L ii) Z iii) i_0 iv) $V(t)$

v) $V_R(t)$ vi) $V_L(t)$

$\therefore i_0 = 10$

$\therefore \omega = 20$

$\rightarrow X_L = \omega L = 0.3(20) = 6 \Omega$

$\rightarrow Z = \sqrt{R^2 + X_L^2} = \sqrt{36 + 36} = 6\sqrt{2}$

$$V_0 = i_0 Z$$

$$= 10(6\sqrt{2}) = 60\sqrt{2}$$

$$V_R(t) = V_{0R} \sin(\omega t + 30^\circ)$$

$$= (i_0 \times R) \sin(\omega t + 30^\circ)$$

$$= (10 \times 6) \sin(20t + 30^\circ)$$

$$V_R(t) = 60 \sin(20t + 30^\circ)$$

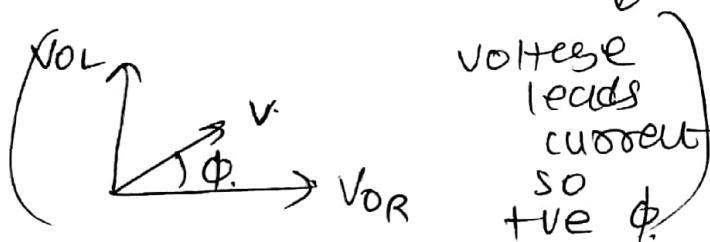
$$V_L(t) = V_{0L} \sin(\omega t + 30^\circ + 90^\circ)$$

$$= (X_L i_0) \sin(20t + 120^\circ)$$

$$V_L(t) = 60 \sin(20t + 120^\circ)$$

$$\phi = \frac{X_L}{R} = \frac{6}{6} = 1 \Rightarrow \phi = 45^\circ$$

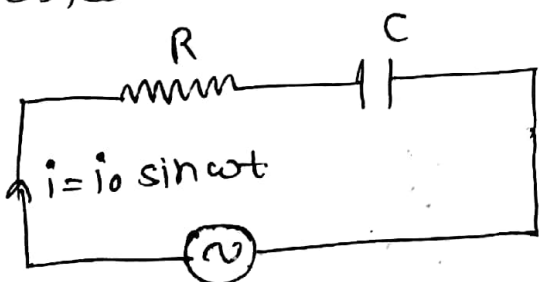
$$\therefore V(t) = 60 \sin(20t + 30^\circ + \phi)$$



voltage leads current so +ve phi

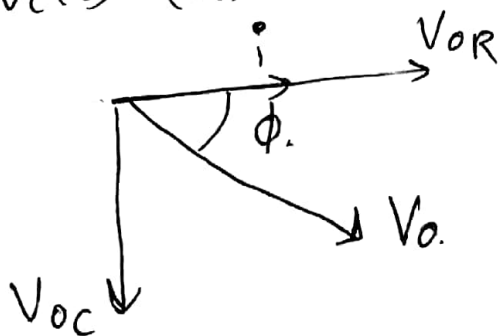
$$\therefore V(t) = 60 \sin(20t + 30^\circ + 45^\circ)$$

② Series C-R circuit :-



$$V_R(t) = (V_0)_R \sin \omega t$$

$$V_C(t) = (V_0)_C \sin(\omega t - \pi/2)$$



$$V_0 = \sqrt{V_{0R}^2 + V_{0C}^2} \quad (1)$$

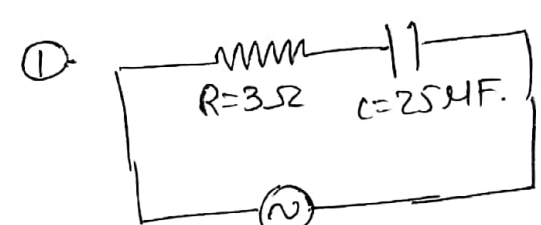
$$V_0 = \sqrt{i_0^2 R^2 + i_0^2 X_C^2}$$

$$V_0 = i_0 \sqrt{R^2 + X_C^2}$$

$$\therefore V_0 = i_0 Z$$

z = impedance = $\sqrt{X_C^2 + R^2}$
 → current leads potential by phi.

$$\tan \phi = \frac{X_C}{R}$$



Find i, X_C , z, i_0 , $i(t)$, $V_R(t)$, $V_C(t)$.

$$\rightarrow X_C = \frac{1}{\omega C} = \frac{1}{10^4 (25 \times 10^{-6})} = 0.04 \times 100 = 4 \Omega$$

$$\rightarrow z = \sqrt{R^2 + X_C^2} = \sqrt{3^2 + 4^2} = 5 \Omega$$

$$\rightarrow i_0 = \frac{V_0}{z} = \frac{10}{5} = 2 \text{ A}$$

$$\rightarrow \phi = \tan^{-1} \frac{X_C}{R} = \tan^{-1} \left(\frac{4}{3} \right) = 53^\circ$$

$$\rightarrow i(t) = i_0 \sin \omega t = 2 \sin(10^4 t + 30^\circ + \phi)$$

$$= 2 \sin(10^4 t + 30^\circ + 53^\circ)$$

$$= 2 \sin(10^4 t + 83^\circ)$$

$$\rightarrow V_R(t) = (V_0)_R \sin(10^4 t + 30^\circ)$$

$$= (i_0 \times R) \sin(10^4 t + 30^\circ)$$

$$= 6 \sin(10^4 t + 30^\circ)$$

$$\rightarrow V_C(t) = V_{0C} \sin$$

$$i(t) = 2 \sin(10^4 t + 83^\circ) \quad (12)$$

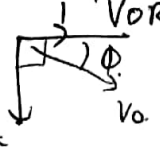
$$\rightarrow V_A(t) = V_{OR} \sin(10^4 t + 83^\circ)$$

$$= i_{OR} \times R$$

$$= 6$$

$$\rightarrow V_C(t) = V_{OC} \sin(10^4 t + 83^\circ - 90^\circ)$$

$$= i_{OC} \times X_C \sin(10^4 t - 7^\circ)$$

$$= 8 \sin(10^4 t - 7^\circ)$$


$$= \frac{V_o i_o \left(\sin^2 \omega t \cos \phi + \frac{2 \sin \omega t \cos \omega t \sin \phi}{2} \right)}{2}$$

$$= \frac{V_o i_o \left(\sin^2 \omega t \cos \phi + \frac{\sin 2 \omega t \sin \phi}{2} \right)}{2}$$

$$= \frac{V_o i_o \left[\overline{\sin^2 \omega t \cos \phi} + \frac{\overline{\sin 2 \omega t \sin \phi}}{2} \right]}{2}$$

$$= \frac{V_o i_o \left[\frac{1}{2} \cos \phi + 0 \right]}{2}$$

$$= \frac{V_o i_o \cos \phi}{2}$$

$$= \frac{V_o}{\sqrt{2}} \frac{i_o}{\sqrt{2}} \cos \phi$$

* POWER IN A.C. :-

→ In current Electricity we use,
 $P = VI$

→ Now; for AC suppose,
 $V = V_o \sin \omega t$
 $i = i_o \sin(\omega t + \phi)$

→ Here; in AC V & i both varies with time, so, P (power) also varies with t .

$\therefore P_{average} = V_{rms} i_{rms} \cos \phi$

$\cos \phi =$ power factor.

(• If not mentioned then assume ~~avg~~ average power.
 • I & $V \rightarrow$ rms i & rms V .)

→ ∴ Instantaneous power 1

$$P(t) = V(t) i(t)$$

$$P(t) = V_o \sin \omega t i_o \sin(\omega t + \phi)$$

* To Find average power

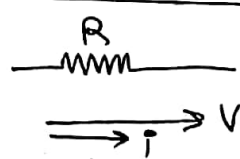
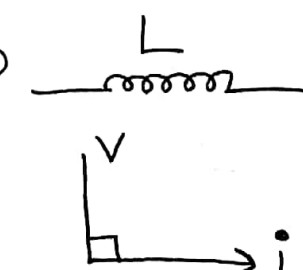
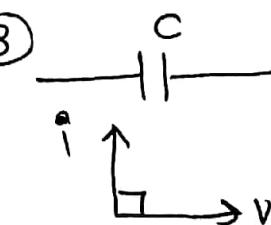
$$P_{avg} = \overline{V_o \sin \omega t i_o \sin(\omega t + \phi)}$$

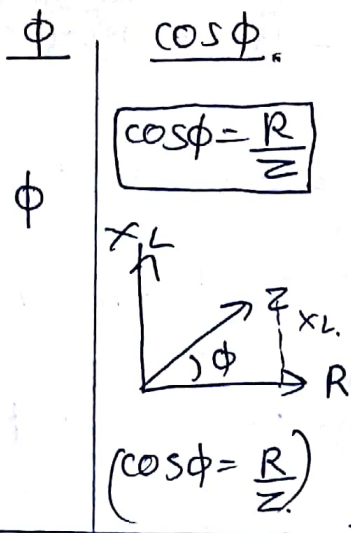
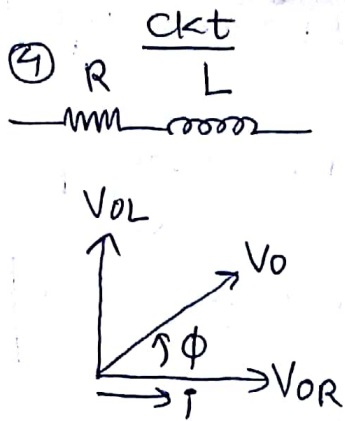
$$= \overline{V_o i_o \sin \omega t \cdot \sin(\omega t + \phi)}$$

(Hint:-
 $\sin(A+B) = \sin A \cos B + \cos A \sin B$)

$$= \overline{V_o i_o \sin \omega t (\sin \omega t \cos \phi + \cos \omega t \sin \phi)}$$

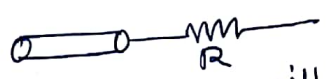
$$= \overline{V_o i_o (\sin^2 \omega t \cos \phi + \sin \omega t \cos \omega t \sin \phi)}$$

| circuit | ϕ | power factor $\cos \phi$. |
|--|--------|----------------------------|
| ①  | 0 | 1 |
| ②  | 90° | 0 |
| ③  | 90° | 0. |



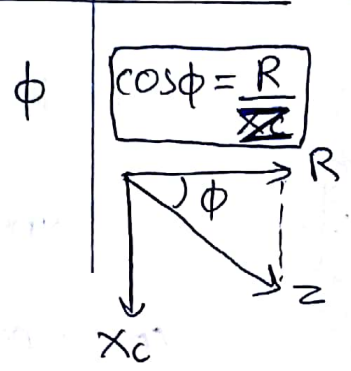
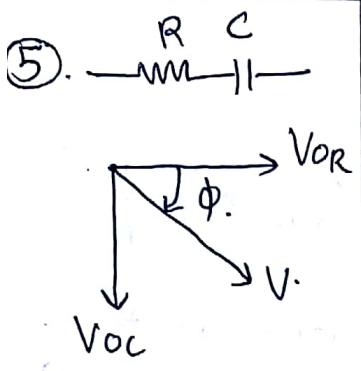
CHOKE COIL

In India household supply is 220V, some device like tube light can't stand this much high voltage
 → To reduce voltage we can use resistor.

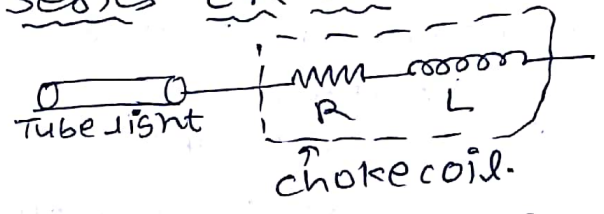


→ But resistor will waste too much power.
 $P = V_{rms} \times i_{rms} \times \cos \phi$ } High.

→ To solve this problem we use a choke coil.



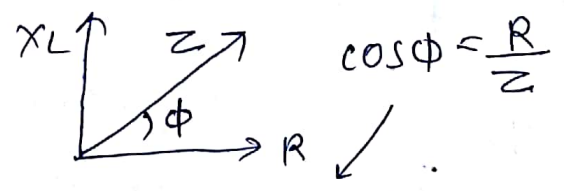
* Series LA ckt :-



choke coil can reduce voltage to tube if its impedance Z is High.

$Z = \sqrt{X_L^2 + R^2}$

• But R should be small.
 $R \downarrow \& Z \uparrow$ i.e. X_L must be \uparrow
 i.e. L must \uparrow .



$\cos \phi = \frac{R}{\sqrt{X_L^2 + R^2}}$

$= \frac{1}{\sqrt{(\frac{X_L}{R})^2 + 1}} \rightarrow \infty$

i.e. $\frac{1}{\infty} \rightarrow 0$

$\therefore \cos \phi \approx 0$ Very small.

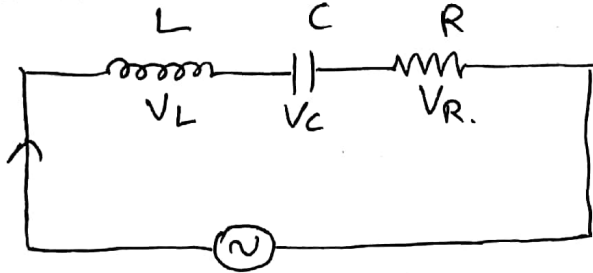
Remember: Always $\cos \phi = \frac{R}{Z}$

i.e. Avg. Power
 $= V_{rms} i_{rms} \cos \phi$
 $= V_{rms} i_{rms} (\frac{R}{Z})$

$P_{avg} = i_{rms}^2 R$
 $P_{avg} = \frac{V_{rms}^2 R}{Z}$ ($\because i_{rms} = \frac{V_{rms}}{Z}$)

* Apparent power = $V_{rms} i_{rms}$ - (3)

SERIES LCR circuit :-



$V = V_m \sin \omega t$

• To Find current we use here two method.

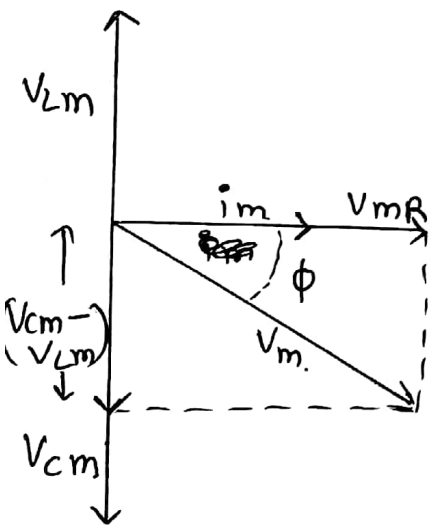
- ① Phasor method
- ② Analytic method.

• PHASOR-diagram solution:-

→ Let; current $i(t)$ is same in all 3 L & C & R

is $i = i_m \sin(\omega t + \phi)$ — ①

where; $\phi =$ phase diff. b/w. source V & i.



→ From phasor diagram :-

$\therefore V_m^2 = V_{Rm}^2 + (V_{cm} - V_{lm})^2$

∴ where;

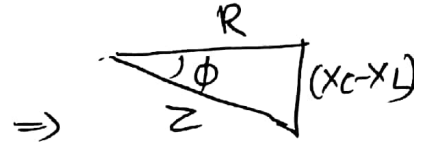
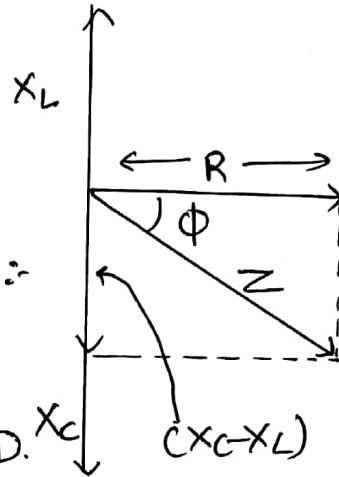
- $V_{Rm} = i_m R$
- $V_{cm} = i_m X_c$
- $V_{lm} = i_m X_L$

$\therefore V_m^2 = i_m^2 R^2 + (i_m^2 X_c - i_m^2 X_L)^2$

$\therefore V_m^2 = i_m^2 [R^2 + (X_L - X_c)^2]$

$\therefore V_m = i_m \sqrt{R^2 + (X_c - X_L)^2}$

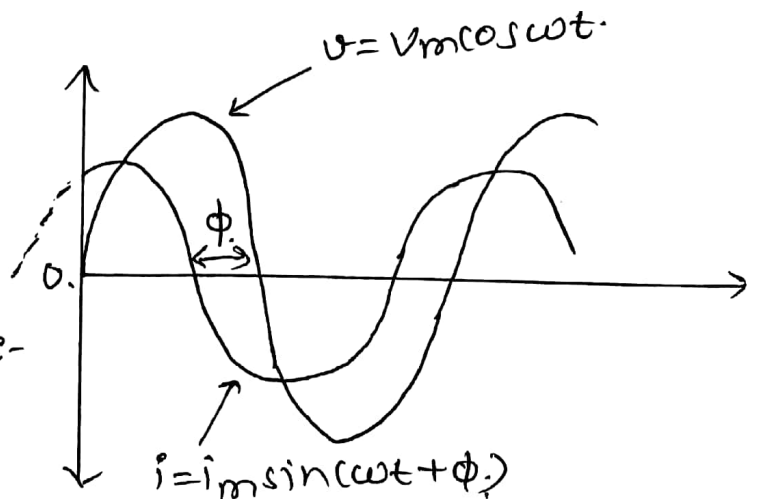
→ impedance $Z = \sqrt{R^2 + (X_c - X_L)^2}$



$\therefore \tan \phi = \frac{X_c - X_L}{R}$

→ Here; $X_c > X_L \Rightarrow \phi = +ve$.
i.e. circuit is capacitive so current leads.

→ If $X_L > X_c \Rightarrow \phi = -ve$
i.e. ckt is inductive & so current lags behind source voltage.



* Analytical solution :-

→ For series LCR AC circuit voltage is given by:

$$\rightarrow V = V_L + V_C + V_R$$

$$V = \left(L \frac{di}{dt} \right) + \frac{Q}{C} + iR$$

→ converting Equation in form of charge q.

$$\therefore V = \left(L \frac{d^2q}{dt^2} \right) + \frac{q}{C} + \frac{dq}{dt} R$$

$$\therefore V_m \sin \omega t = L \frac{d^2q}{dt^2} + \frac{q}{C} + \frac{dq}{dt} R \quad \text{--- (2)}$$

→ This Eq. is like forced damped Harmonic oscillator. (3)

→ let's assume a solution of Eq. (2) } $q = q_m \sin(\omega t + \theta)$ (3)

$$\therefore \left[\frac{dq}{dt} = q_m \omega \cos(\omega t + \theta) \right] \left[\frac{d^2q}{dt^2} = -q_m \omega^2 \sin(\omega t + \theta) \right]$$

(4) (5)

→ Putting (3), (4), (5) in (2) :-

$$\therefore V_m \sin \omega t = L \left(-q_m \omega^2 \sin(\omega t + \theta) \right) + \frac{q_m \sin(\omega t + \theta)}{C} + \frac{q_m \omega \cos(\omega t + \theta)}{R}$$

$$\therefore V_m \sin \omega t = q_m \omega \left[-(\omega L) \times \sin(\omega t + \theta) + \left(\frac{1}{\omega C} \right) \sin(\omega t + \theta) + \frac{R}{\omega} \cos(\omega t + \theta) \right]$$

$$V_m \sin \omega t = q_m \omega \left[\frac{-X_L}{\sin(\omega t + \theta)} + X_C \sin(\omega t + \theta) + R \cos(\omega t + \theta) \right]$$

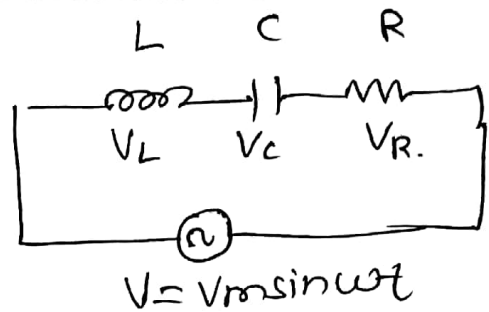
$$\left[\begin{array}{l} X_L = \omega L \\ X_C = \frac{1}{\omega C} \end{array} \right] \left[\begin{array}{l} \sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta \\ \sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta \\ \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta \\ \cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta \end{array} \right]$$

$$\therefore V_m \sin \omega t = q_m \omega \left[(X_C - X_L) \sin(\omega t + \theta) + R \cos(\omega t + \theta) \right]$$

$$\therefore \Rightarrow \frac{V_m}{Z} \sin \omega t = q_m \omega \left[\frac{X_C - X_L}{Z} \sin(\omega t + \theta) + \frac{R}{Z} \cos(\omega t + \theta) \right]$$

$$\frac{V_m}{Z} \sin \omega t = q_m \omega \left[\sin \phi \cdot \sin(\omega t + \theta) + \cos \phi \cdot \cos(\omega t + \theta) \right]$$

$$\left(\because \cos \phi = \frac{R}{Z} \text{ \& } \sin \phi = \frac{X_C - X_L}{R} \right)$$



(Above form is like
 $\therefore \cos(\omega t + \theta) \cos \phi + \sin(\omega t + \theta) \sin \phi = \cos(\omega t + \theta - \phi)$)

$$\therefore \frac{V_m}{Z} \sin \omega t = q_m \omega [\cos(\omega t + \theta - \phi)]$$

$$V_m \sin \omega t = (q_m \omega) Z \cos(\omega t + \theta - \phi)$$

$$\therefore V_m \sin \omega t = i_m Z \cos(\omega t + \theta - \phi)$$

$$(\because i_m = q_m \omega)$$

$$\therefore \text{we have: } V_m = i_m Z$$

$$\therefore \sin \omega t = \cos(\omega t + \theta - \phi)$$

$$\therefore \cos\left(\frac{\pi}{2} - \omega t\right) = \cos(\omega t + \theta - \phi)$$

$$\therefore \cos(\omega t - \frac{\pi}{2}) = \cos(\omega t + \theta - \phi)$$

From above Eqn

$$\therefore \frac{-\pi}{2} = \theta - \phi$$

$$\therefore \boxed{\theta = \frac{-\pi}{2} + \phi} = \boxed{\phi - \frac{\pi}{2} = 0} \quad \text{--- (6)}$$

~~which is the solution of~~

$$\rightarrow \text{Put (6) in (3): } \boxed{q = q_m \sin(\omega t + \phi - \frac{\pi}{2})}$$

this is the solⁿ of Eqn. (2).

$$\therefore \text{current in ckt is :- } i = \frac{dq}{dt} = q_m \omega \cos(\omega t + (\phi - \frac{\pi}{2}))$$

$$= q_m \omega \cos\left[\frac{\pi}{2} - \omega t - \phi\right]$$

$$= -q_m \omega \sin(\omega t - \phi)$$

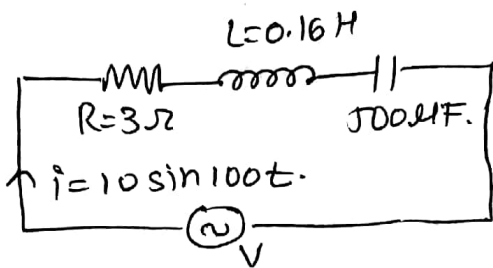
$$i = q_m \omega \sin(\omega t + \phi)$$

$$\therefore \boxed{i = i_m \sin(\omega t + \phi)} \rightarrow \star$$

$$\text{where: } i_m = \frac{V_m}{Z} = \frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}}$$

$$\& \phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$$

Q.1)



Find.

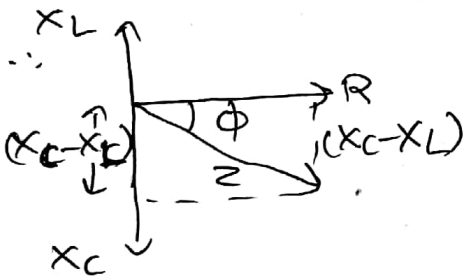
$X_L, X_C, Z, \phi, V_o, V_L(t), V_R(t), V_C(t), \text{Power factor.}$

Ans: ① $X_L = \omega L = 100(0.16) = 16 \Omega$

② $X_C = \frac{1}{\omega C} = \frac{1 \times 10^6}{100 \times 500} = 0.2 \times 10^2 = 20 \Omega$

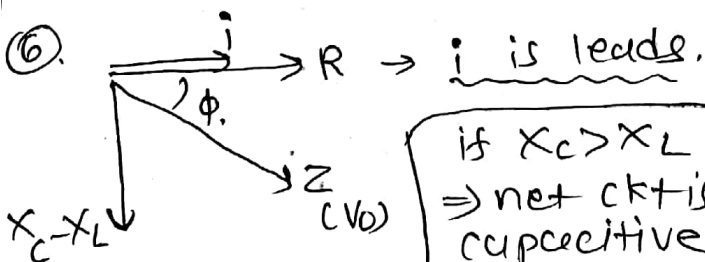
③ $Z = \sqrt{R^2 + (X_C - X_L)^2} = \sqrt{9 + (20 - 16)^2} = 5 \Omega$

Here, $X_C > X_L$



④ $\phi = \tan^{-1} \frac{(X_C - X_L)}{R} = \tan^{-1} \frac{4}{3} = 53^\circ$

⑤ $V_o = i_o Z = 10(5) = 50$



if $X_C > X_L$
 \Rightarrow net ckt is capacitive
 i.e. $i(t)$ leads $v(t)$.

$v(t) = V_o \sin(100t - \phi)$

$v(t) = 50 \sin(100t - 53^\circ)$

⑦ $V_R(t) = (V_o)_R \sin(100t) = (i_o \times R) \sin(100t) = (30) \sin(100t)$

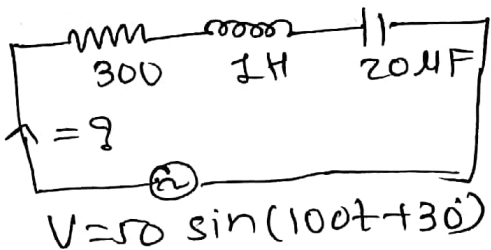
⑧ $V_L(t) = (V_o)_L \sin(100t + 90^\circ) = i_o \times X_L \sin(100t + 90^\circ) = 160 \sin(100t + 90^\circ)$

⑨ $V_C(t) = (V_o)_C \sin(100t - 90^\circ) = i_o \times X_C \sin(100t - 90^\circ) = 200 \sin(100t - 90^\circ)$

⑩ $\cos \phi = \frac{R}{Z} = \frac{3}{5}$

⑪ Avg. Power = $V_{rms} i_{rms} \cos \phi = \frac{V_o}{\sqrt{2}} \frac{i_o}{\sqrt{2}} \cdot \frac{3}{5} = \frac{50 \times 10 \times 3}{2 \times 5} = 150 \text{ W}$

Q.2)



Find $X_L, X_C, Z, \phi, i_o, i(t), V_R(t), V_L(t), V_C(t)$ power factor & power.

Ans. $X_L = 100 \Omega, X_C = 500 \Omega$

$X_C > X_L \rightarrow$ capacitive ckt.

$Z = 500 \Omega$

$\phi = \frac{X_C - X_L}{R} = \frac{400}{300} = 53^\circ$

$\therefore i_o = 0.1 \text{ A}$

$i(t) = 0.1 \sin(100t + 30^\circ + 53^\circ)$

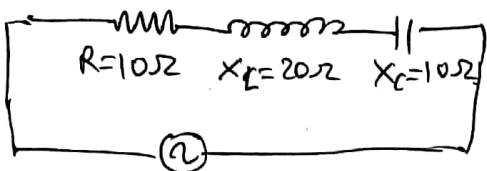
$V_R(t) = 30 \sin(100t + 83^\circ)$

$V_L(t) = 10 \sin(100t + 173^\circ)$

$V_C(t) = 50 \sin(100t - 7^\circ)$

$\left. \begin{aligned} \cos \phi &= \frac{3}{5} \\ P_{avg} &= 105 \text{ W} \end{aligned} \right\}$

Q.3)



$$E = 20\sqrt{2} \sin(100t + 30^\circ)$$

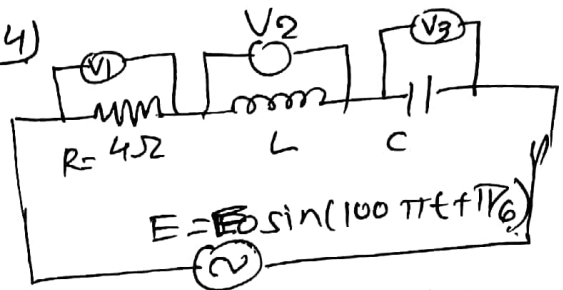
Find $Z, \phi, i_0, i_{rms}, (V_{rms})_R, (V_{rms})_L, (V_{rms})_C$

Ans → Here: $X_L > X_C \Rightarrow$ $ck + j$ Inductive.



$$\begin{aligned} Z &= \sqrt{R^2 + (X_L - X_C)^2} \\ &= \sqrt{10^2 + 10^2} = 10\sqrt{2} \end{aligned}$$

Q.4)



$$E = 10 \sin(100\pi t + \pi/6)$$

Reading of voltmeters:-

$$V_1 = 40V, V_2 = 40V, V_3 = 10V$$

Find ① i_0, i_{rms} ② E_0 ③ $L \& C, i(t)$

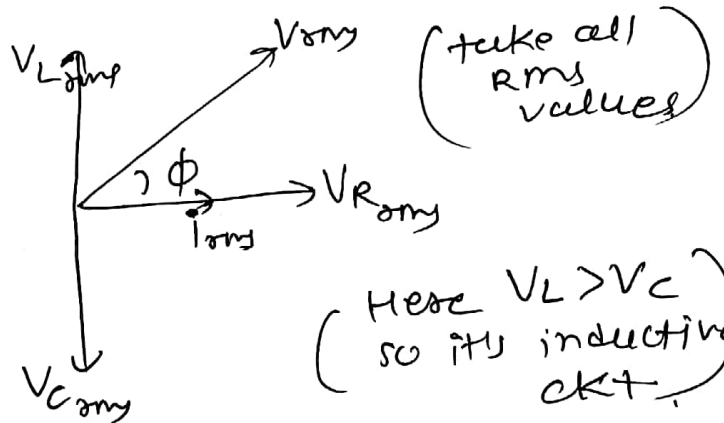
→ these voltmeter measured V_{rms} in R & L, & C resply.

$$\rightarrow \text{i.e. } (V_{rms})_R = 40V = i_{rms} R$$

$$\therefore i_{rms} = 10A$$

$$i_0 = i_{rms} \sqrt{2} = 10\sqrt{2} A$$

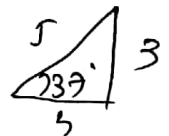
$$\rightarrow i(t) = 10\sqrt{2} \sin(100\pi t - \dots)$$



Here $V_L > V_C$ so it's inductive $ck + j$.

$$\therefore \phi = \tan^{-1} \left(\frac{V_L - V_C}{V_R} \right) = \frac{30}{40} = \frac{3}{4}$$

$$\therefore \phi = 37^\circ$$



$$i(t) = 10\sqrt{2} \sin(100\pi t - 37^\circ + \pi/6)$$

$$= 10\sqrt{2} \sin(100\pi t - 37^\circ + 30^\circ)$$

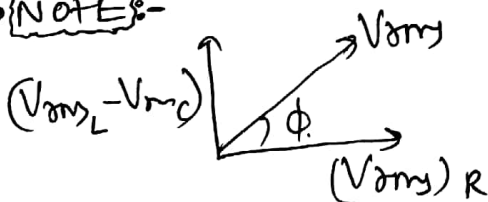
$$i(t) = 10\sqrt{2} \sin(100\pi t - 7^\circ)$$

$$\begin{aligned} (V_{rms})^2 &= V_R^2 + (V_L - V_C)^2 \\ &= \sqrt{40^2 + (30)^2} = 50 \end{aligned}$$

$$V_{rms} = 50 \Rightarrow V_0 = V_{rms} \sqrt{2} = 50\sqrt{2} = E_0$$

$$V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{20\sqrt{2}}{\sqrt{2}} = 20V$$

NOTE:-



$$\begin{aligned} \therefore V_{rms} &= \sqrt{(V_{rms})_R^2 + (V_{rms})_L - (V_{rms})_C^2} \\ &= \sqrt{(10\sqrt{2})^2 + (20\sqrt{2} - 10\sqrt{2})^2} \\ &= \sqrt{(10\sqrt{2})^2 + (10\sqrt{2})^2} \\ &= \sqrt{2000 + 2000} \\ &= 20V \end{aligned}$$

For L&C

$$\therefore (V_{rms})_L = i_{rms} \times X_L$$

$$\therefore 40 = 10 \omega L$$

$$\therefore L = \frac{4}{\omega} = \frac{4}{100\pi} = \frac{1}{25\pi} \text{ H.}$$

$$\rightarrow V_{rms}_C = i_{rms} \times X_C$$

$$\therefore 10 = 10 \frac{1}{\omega C}$$

$$\therefore C = \frac{1}{\omega} = \frac{1}{100\pi} \text{ Farad}$$

Resonance in L-C-R series AC circuit:-

$$\bullet \ i_m = \frac{V_m}{Z} = \frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}}$$

• when $X_L = X_C$ (or $V_L = V_C$)
 $\Rightarrow \omega_0$ is called as Resonant frequency.

\rightarrow And, resonance takes place series

\rightarrow when $X_L = X_C$ or $f_0 = \frac{1}{2\pi\sqrt{LC}}$

$$= \omega_0 L = \frac{1}{\omega_0 C}$$

$$\therefore \omega_0^2 = \frac{1}{LC}$$

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}} \text{ Resonance freq.}$$

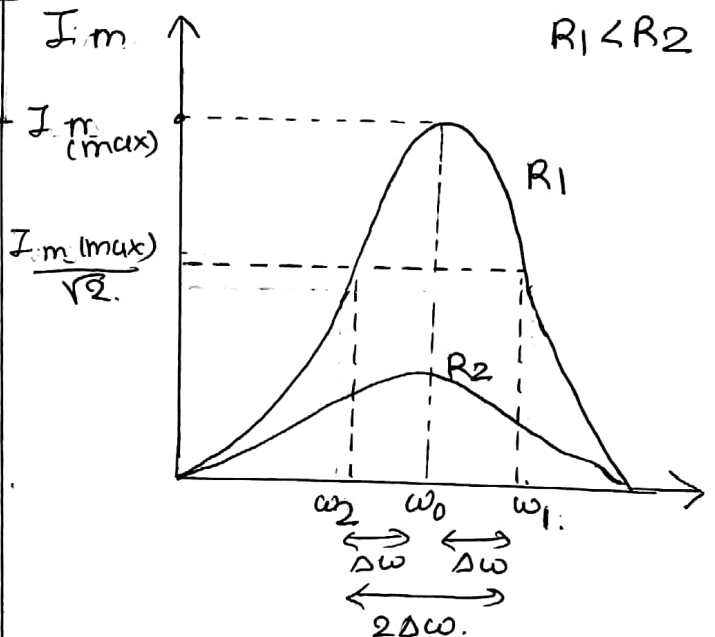
\rightarrow At resonance freq, current amplitude is max.
 $(i_m = V_m/R)$ ($\because Z = R$)

" For some value of ω , of given voltage, \Rightarrow we get maximum value of current i_{rms} is called as series Resonance."

USES :- (Resonance circuit)
 \rightarrow in tuning mechanism of a radio or a TV set

Note :- Resonance is exhibited by a circuit only if both L&C are present.
 i.e. RL&RC can't have resonance.

* SHARPNESS OF RESONANCE



• when $\omega = \omega_0 = \frac{1}{\sqrt{LC}}$, So current $(i_m)_{max} = \frac{V_m}{R}$ which is maximum.

• Here; $\omega_1 = \omega_0 + \Delta\omega$
 $\omega_2 = \omega_0 - \Delta\omega$

• Band width :- $\omega_1 - \omega_2 = 2\Delta\omega$ is called as Bandwidth.

• Sharpness of Resonance : $= \frac{\omega_0}{(2\Delta\omega)} = \frac{R \cdot f}{B.W.}$

$\rightarrow B.W \downarrow \Rightarrow$ sharpness \uparrow .

Half Power B.W :-

→ For any value of ω other than ω_0 , current becomes $\frac{1}{\sqrt{2}}$ times of its maximum value.

⇒ power dissipated by ckt becomes half.

⇒ $\Delta \omega$ is called as (2 Δ) Half power B.W.

($\because \omega_1 - \omega_2 = \text{Half P. B.W.}$)

* Q-Factor (Quality factor) :-

• Sharpness of curve is known by Q Factor.

→ $Q = \frac{\omega_0}{\Delta \omega} = \frac{\omega_0}{2\Delta \omega}$ — ①

→ $\Delta \omega = \frac{R}{2L}$ — ② (deriv. NERT p. 250)

\therefore ② → ① ⇒ $Q = \frac{\omega_0}{2(\frac{R}{2L})} = \frac{\omega_0 L}{R}$

$Q = \frac{\omega_0 L}{R}$ or $Q = \frac{1}{\omega_0 CR}$

• sharp the resonance ⇒ selectivity of ckt is good.

• less sharp ⇒ not good. wide

ie $(R \downarrow, L \uparrow) \Rightarrow Q \uparrow \Rightarrow \text{selectivity} \uparrow$

$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR} = \frac{1}{R} \sqrt{\frac{L}{C}}$

Note :-

• Q too high → sharp resonance & small b.w. & large $\Delta \omega$

• $(i_{rms})_{max} \rightarrow (i_{rms})_{max} \rightarrow \frac{(i_{rms})_{max}}{\sqrt{2}}$
 \downarrow
 \downarrow
 $\frac{P_{max}}{2} \rightarrow P_{max} \rightarrow \frac{P_{max}}{2}$

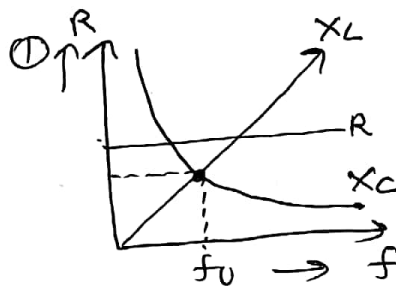
• $P_{max} = \frac{(i_{rms})_{max}^2 R}{2} = (i_{rms})_{max}^2 R$

• Q-Factor = $\frac{\text{Reso. freq. } (\omega_0)}{\text{B.W. } (\Delta \omega)}$

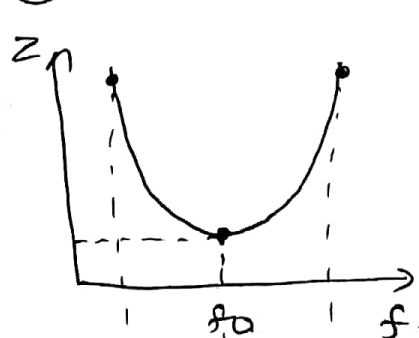
GRAPHS :-

$X_L = \omega L = 2\pi f L$

$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$



② $Z \rightarrow f$

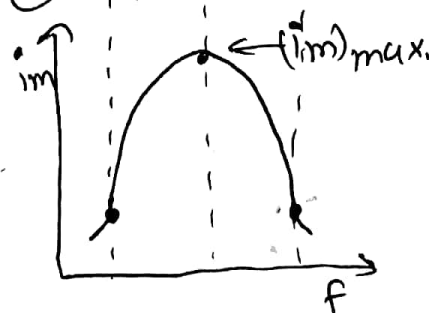


$Z = \sqrt{R^2 + (X_L - X_C)^2}$
 $= \sqrt{R^2 + (2\pi f L - \frac{1}{2\pi f C})^2}$

i.e. f small ⇒ Z large

f large ⇒ Z large

③ $i_{rms} \rightarrow f$



At resonance power is also max

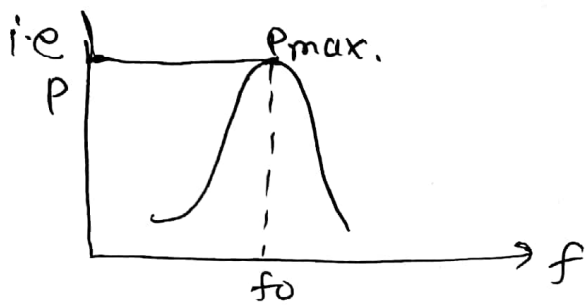
$$P = V_{rms} i_{rms} \cos\phi \leftarrow \text{# for } R$$

$$= \frac{V_{rms}}{\sqrt{2}} \frac{i_{rms}}{\sqrt{2}} (1)$$

$$P = \frac{(i_{rms})_{max}^2 R}{2} \times (i_{rms})_{max}$$

$$P = \frac{(i_{rms})_{max}^2}{2} \times R$$

$$P = (i_{rms})_{max}^2 R$$



- ① At resonance $Z_{min} = R$,
 $i_{0max} = V_0/R$,
 i & V are in phase.
 (∵ it's become purely R. ckt)
 $P \rightarrow max = i_{rms}^2 R = \frac{i_{rms}^2 \times R}{2}$

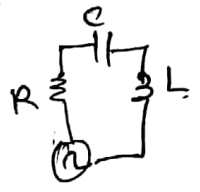
② Reso. freq. $\omega_0 = \frac{1}{\sqrt{LC}}$, $f_0 = \frac{1}{2\pi\sqrt{LC}}$
 $\therefore X_L = X_C$

- ✓ The resonant freq. can be changed by changing values of C & L .
- ✓ There is no effect on f_0/ω_0 by value of R .
- ✓ R affects $(i_0)_{max}$.



Radio Tuning :-

- Antenna of radio catches (fig. d) all signals (e.m.s) from atmosphere.
- To select some channel,
 \rightarrow Rotate Radioknob.
 \downarrow
 adjust value of C or L
 \downarrow
 so, we adjust reso. freq. f_0 .

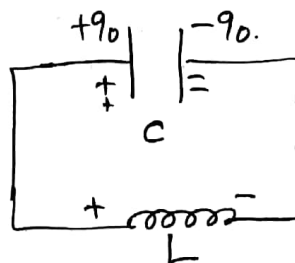


Power in AC circuit :- (NCERT p.no. 252)

L-C OSCILLATION :-

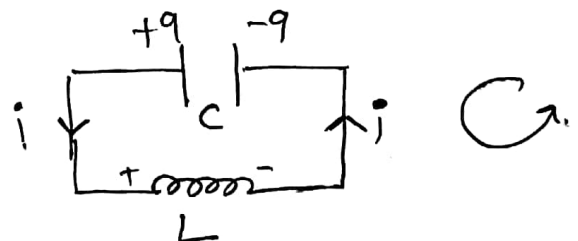
- capacitor can store Electrical Energy & Inductor \rightarrow Mag. E.
- when fully charged C is connected to inductor, the charge on C & i in ckt starts electrical oscillations.
- consider cap. is fully $\left. \begin{array}{l} \text{charged } \{ q_m \\ \text{\& connected} \end{array} \right\}$ to inductor

($t=0 \Rightarrow q=q_m$)



- when ckt completed q starts \downarrow & i starts \uparrow .

at time $t \Rightarrow t$, $q=q$, $i=i$.



Now: applying KVL:

$$\therefore +V_C - V_L = 0$$

$$\therefore \boxed{\frac{q}{C} - \left[L \frac{di}{dt} \right] = 0} \quad \text{--- (1)}$$

Here: $\boxed{i = -\frac{dq}{dt}}$ (charge decreases) --- (2)

→ Put (2) in (1):

$$\therefore \frac{q}{C} - L \left(-\frac{d^2q}{dt^2} \right) = 0$$

$$\therefore \frac{q}{C} + L \frac{d^2q}{dt^2} = 0$$

$$\therefore L \boxed{\frac{d^2q}{dt^2} + \frac{q}{LC} = 0} \quad \text{--- (3)}$$

→ comparing above Eqn. with Eqn of S.H. oscillator

$$\boxed{\frac{d^2x}{dt^2} + \omega_0^2 x = 0}$$

→ we get; $\boxed{\omega_0^2 = \frac{1}{LC}}$

$$\therefore \omega_0 = \frac{1}{\sqrt{LC}}$$

→ Now: solution of Eqn (3) is of the form

$$\boxed{q = q_m \cos(\omega_0 t + \phi)}$$

→ where: $q_m = \text{Max value of } q$

$$t=0 \Rightarrow q = q_m \text{ i.e. } \phi = 0$$

i.e. $\boxed{q = q_m \cos(\omega_0 t)}$

$$\rightarrow i = -\frac{dq}{dt} = -\frac{d}{dt}(q_m \cos \omega_0 t)$$

$$= -q_m \omega_0 (-\sin \omega_0 t)$$

$$\boxed{i = q_m \omega_0 \sin \omega_0 t}$$

$$\boxed{i = i_m \sin \omega_0 t}$$

where: $i_m = q_m \omega_0$

NCEERT- p.no. 256.

Q.1 Analogies b/w Mechanical & Electrical Quantities

Q.2 Transformer. (p.no. 259)

Q.3 In actual transformer, due to which reasons Energy will be losses? (p.261)

Q.4 In LC ckt, sum of Energy stored in ckt is const. (p.no. 259) m7mp.